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APPLICATIONS OF AN INTERACTIVE BALANCING PROCEDURE FOR GAS TURBINES AND OTHER TURBOMACHINERY

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ABSTRACT

Least squares balancing methods have been applied for many years to reduce vibration levels of turbomachinery. This approach yields an optimal configuration of balancing weights to reduce a given cost function. However, in many situations, the cost function is not well-defined by the problem, and a more interactive method of determining the effects of balance weight placement is desirable. An interactive balancing procedure is outlined and implemented in an Excel spreadsheet. The usefulness of this interactive approach is highlighted in balancing case studies of a GE LM5000 gas turbine and an industrial fan. In each case study, attention is given to practical aspects of balancing such as sensor placement and balancing limitations.

BALANCING NEEDS AND APPLICATION

Reduction of vibration is a goal of rotating machinery operators based on the realization that excessive vibration implies transmitting dynamic loads through the bearings that can lead to failure or foreshortened life. Other dynamic issues are known to arise from excessive vibration such as fatigue of supporting structure, fretting at support joints, and noise hazard.

Rotor imbalance is usually the most significant cause of excessive vibration; however, several other sources are likely depending on the design and application of the machine. Historically, bearing whirl instability has plagued development of higher speed rotors that use hydrodynamic bearings, but rotor-dynamic technology developments have produced bearings with tilting pad design, pressurized hydrostatic bearings, and special loading features such as pressure dam and elliptical form bearings that are more resistant to whirl instabilities, but still problems persist. Advanced high pressure machinery continues to be plagued by hydrodynamic instabilities with sources in the fluid/structure interactions of seals and compressor impellers. Rolling element bearings usually produce vibration related to the interactions of the rollers (balls) with the races; these levels are usually low except that spalling defects create strong impulses at ball pass and ball rotation frequencies, which signal that the remaining life of the bearing is probably short. Rotor misalignment is another common source of vibration and usually results in a two times rotor speed vibration component. To be successful in reducing vibration, its source needs to be clearly identified; fortunately, spectral analysis can be used to differentiate between the different vibration sources. Typically, there may be more than one source, and each needs to be addressed separately. Table 1 presents the sources of vibration typically found in turbomachinery and the distinguishing features in the spectrum. Vibration due to imbalance will not only be exactly consistent with rotor speed, it will exhibit a stable phase relationship to rotation as well. It is this phase consistency that allows imbalance errors to be corrected.

Table 1. Typical Vibration Sources in Turbomachinery

Vibration Source	Frequency	Comment
Unbalance	Rotor speed	Consistent phase at a given speed
Misalignment	2X Rotor speed	1X Rotor speed sometimes seen
Bearing whirl	½ X Rotor speed	Frequency may vary slightly less than ½ X
Rolling element bearing race defect	Ball or roller pass frequency on affected race	Plus multiples
Rolling element bearing element defect	Ball or roller rotation frequency	Plus multiples
Rotor/Bearing instability	Sub-synchronous vibration related to critical speed	

Vibration Source	Frequency	Comment
Rotating stall of compressors	Typically 20% to 50% of rotor speeds (axial compressors)	Plus multiples
Rotor/case rubbing	Approximately 3X rotor speed	
Bearing looseness	Harmonics of rotor speed (2X, 3X, etc.)	Possible sub-harmonics (X/2, X/3, etc.)

For those interested, the Schenck website on “100 years of balancing technology” [1] provides an enjoyable review of balancing methods from their perspective. A notable quote is “Nowadays we find it hard to believe that balancing of a steam turbine rotor took three to four weeks of hard manual labor in the early days of industrialization”. In the early years, static balancing of rotor components on knife edges was commonly used to balance disk like rotors, and the practice is still used to pre-balance individual disks that are then preferentially assembled into built up rotors. H. Martinson [2], a Canadian engineer, was granted what was probably the first patent for a balancing machine in 1870. The rotor was mounted on soft springs so that it spun about its center of mass; the high spot was marked by moving a piece of chalk gradually towards the rotating rotor. Interestingly, a colleague, Alex Lifson [3], recalls using the same procedure in the Soviet Union before immigrating to the U.S. He was able to determine the position of the unbalance with some degree of accuracy. Cut and try procedures and experience preceded mathematical methods, but the influence coefficient method gained in popularity once vibration phase could be reckoned.

BALANCING METHODOLOGY

The influence coefficient method for balancing requires accurate measurement of vibration vectors (amplitude and phase) for an initial condition (A) and after a known trial weight vector (W_t) is applied (B). The influence coefficient vector is calculated for a given location on the machine and at a specific operating condition (speed, temperature, load, etc.) as

$$\alpha = \frac{B - A}{W_t}. \tag{1}$$

Vibration at this location and condition can be calculated for any other balance weight vector (W) as

$$C = A + \alpha(W) \tag{2}$$

and the correction weight required to reduce the vibration to zero becomes

$$W_c = \frac{-A}{\alpha}. \tag{3}$$

While this correction theoretically reduces the vibration to zero, it only affects the specific location of the measurement, applies to only one balance weight and any measurement errors of vibration vectors A and B, and trial weight magnitude or placement errors are compounded. Multiple balance planes can be accommodated by expanding this equation which sets to zero the vibration magnitude at as many locations or speed conditions as balance planes are available. This is known as the exact point balancing procedure [4, 5]:

$$[W_c] = -[\alpha]^{-1}[A]. \tag{4}$$

The 60s and 70s was a prolific era of balancing publications [6-28] the contribution by Goodman [6] was most significant in that it cast the balancing problem into a least squares minimization regime. This approach allowed vibration at multiple locations and at multiple speed conditions (corresponding to critical speeds) to be minimized with a lesser number of balance correction planes. Goodman’s solution for optimum balance weight vector proceeded by defining a residual vibration for each point and condition as

$$\varepsilon_m = A_m + \sum_{n=1}^{N} \alpha_{mn} W_n . \tag{5}$$

In Eq. (5), N is the number of balance planes and $m = 1, \ldots, M$, where M is the number of vibration readings. The multi-plane balance weight array for least squares minimization of vibration is found by letting

$$S = \sum_{m=1}^{M} \left|\varepsilon_m\right|^2 \tag{6}$$

and minimizing S by setting

$$\frac{\partial S}{\partial W_n} = 0 \tag{7}$$

for all N balance planes. The resulting correction balance weight array is

$$[W_c] = -\left\{[\alpha]^T[\alpha]\right\}^{-1}[\alpha]^T[A]. \tag{8}$$

This method has been widely applied to balancing of large power generation turbines with many rotor critical speeds and many bearing points to minimize vibration [7-13]. It provides good results as long as the vibration effects at all points and speeds are equal. Solution of Eq. (8) requires the computational power of a graphing calculator or laptop to address multi-plane problems, but for a single-plane case, Eq. (8) simplifies to two equations that can be evaluated with a relatively simple function hand calculator:

$$W_x = \frac{-\sum_m (\alpha_{mx} A_{mx} + \alpha_{my} A_{my})}{\sum_m (\alpha_{mx}^2 + \alpha_{my}^2)} \qquad (9)$$

$$W_y = \frac{\sum_m (\alpha_{my} A_{my} + \alpha_{mx} A_{mx})}{\sum (\alpha_{mx}^2 + \alpha_{my}^2)} \qquad (10)$$

Determining influence coefficients requires a minimum of one weight change and run to speed for each balance plane, and often, turbines must be held at loaded conditions for sufficient time to allow thermal transients to stabilize. This can be expensive in terms of fuel costs for large power production turbines. To mitigate some of this cost, balancing professionals have used past experience from similar rotors, predictions from rotor dynamics simulations, and impact testing [14] to get reasonable estimates to start the balancing process. Influence coefficients often experience significant variance from run to run and for data recorded from the same model rotor; this is because rotor balance is often affected by thermal gradients in the rotor that causes slight distortions and unintended shifts in the center of mass rotation. Larsson [15] developed regression techniques to minimize the influence coefficient variance.

Balancing to minimize vibration is the usual goal, but the effect of reduced vibration is to minimize alternating loads or hydraulic pressures that results in bearing fatigue damage. A more direct measurement would be bearing pressure in a hydrodynamic bearing as applied by Christensen, et al [16].

One of the advantages of the least squares method for single plane balance is that it compensates for errors inherent in a single point vibration measurement by effectively averaging the measurements from several different points. Goodman and others recognized that it is more important to minimize vibration at more critical locations and conditions than at others; e.g., rather at running speed than at a vibration transient as the rotor passes through a critical speed. Several approaches of constrained least squares or weighting the results with multipliers have been used that allow judgment to weigh in [17-28].

Some judgment approaches have been successful, others have not been, or they were difficult to apply widely. For the problem of correcting the vibration of an operating machine, several practical issues need to be considered:

1. Influence coefficient errors are usually greater in passing through a critical speed than at a steady operating speed, because the phase and magnitude of vibration are changing rapidly.
2. Rotor response to unbalance magnitude is often nonlinear affecting influence coefficient calculation.
3. It is difficult to determine the appropriate sensitivity for different operating conditions and at different locations before the balancing attempt
4. Time is a factor to minimize with operating equipment; because of fuel costs for off demand operation or lost production to shutdown, a one-shot balance procedure is much preferred.
5. Usually, only a single balance plane is available for corrections or is sufficient for correcting vibration for a single speed condition

For these reasons, the authors developed an interactive balance procedure for single-plane balancing that can be used with a laptop computer. The procedure simply applies Eq. (2) to all vibration points and conditions and allows the projection of correction vectors from the base line condition. The balance weight vector can be moved interactively on the screen. An illustration of the method is presented in the following case studies.

CASE STUDY 1: GE LM5000 GAS TURBINE

This section describes an interactive balancing experience of a GE LM5000 gas turbine installed in a power generation facility. A photograph of the turbine (on a stand) is shown in Fig. 1, and a schematic of the turbine with low-pressure rotor with the low-pressure compressor (LPC) and low-pressure turbine (LPT) locations indicated is shown in Fig. 2. This unit had been experiencing vibration problems at the LPT, and balancing was recommended to reduce vibration levels.

Aero-derivative engines are not designed for routine balancing at site, but a balance plane at the LM5000 LPC is accessible for balance weight placement. Installation of balance weights at the turbine end requires partial disassembly of the gas turbine which requires the facilities of a service depot, resulting in transportation and depot fees and time out of service. A review of balance results to minimize LPC vibrations revealed that vibration vectors at LPT were also affected, but by only about 1/3rd as much. While it was recognized that the effect of LPC balance corrections would be attenuated by the vibration transmission path through a long slender fan shaft or through the casing from LPC to the LPT, it was decided to make an attempt to use this approach to avoid the high cost of balancing the LPT at depot. The objective of this effort was to minimize LPT vibrations by controlling balance weights at the compressor end while keeping LPC vibration levels low as well.

Figure 1. GE LM5000 Gas Turbine on Stand

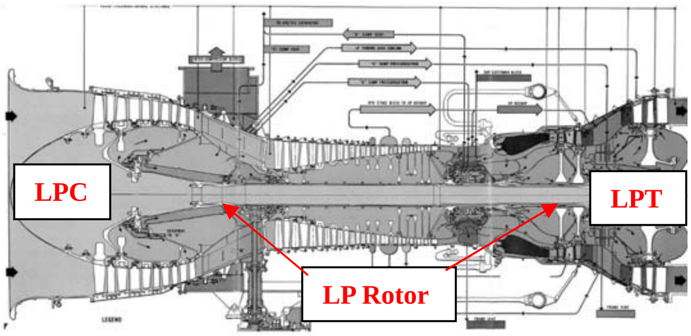

Figure 2. Schematic of GE LM5000 Gas Turbine (from GEAE)

The interactive balancing method described in this paper was used to simultaneously evaluate the effects of placing balance weights on LPC and LPT vibrations. Fig. 3 shows the anticipated balance weight placement that was calculated using influence coefficients from the earlier LPC balancing runs. This balance weight placement was predicted to reduce both LPC and LPT vibrations. The interactive action is accomplished by left-clicking on the "Cor Wt" symbol and moving it left-right and/or up-down to visualize the effects of correction weight placement on the vibration vectors.

Balancing was performed by placing a smart laser key phase sensor at the compressor inlet and measuring vibration levels from installed velocity probes from the gas turbine's monitoring system. The key phase sensor was placed just next to the compressor inlet and acquired a signal once per revolution from a piece of reflective tape on the compressor nose cone. Data from the key phase sensor and velocity probes were acquired using an Alta Solutions AS-1050 data acquisition system, and Machinery Analyzer software was used to generate Bode and Nyquist plots of the vibration data.

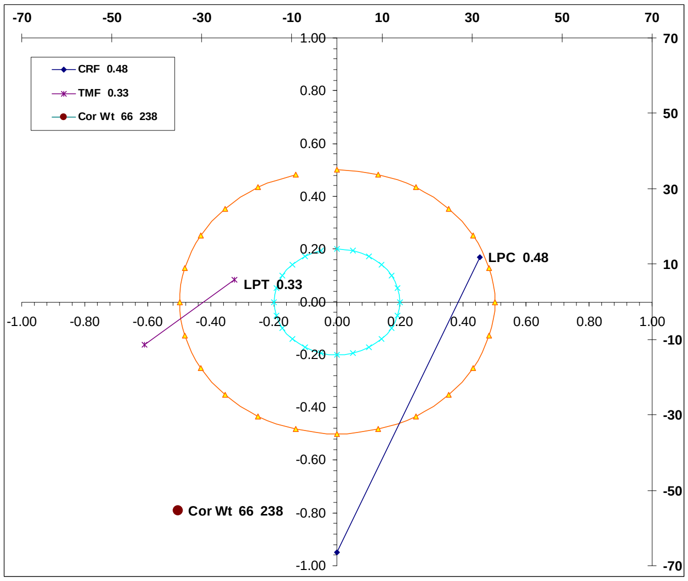

Figure 3. Predicted Vibration Levels for Initial Weight Vector

Vibration data acquired at a typical running speed (approx 3800 rpm) were used to calculate influence coefficients for the LPC and LPT vibrations based on balance weight placement at the LPC. A Bode plot showing the magnitude and phase of vibrations captured at the LPC around the running speed is shown in Fig. 4 for the initial run. The Bode plot for the LPT from the initial run is shown in Fig. 5. The magnitude and phase at running speed for all runs are listed in Table 2, and influence coefficients calculated by comparing each run with the others are shown in Table 3.

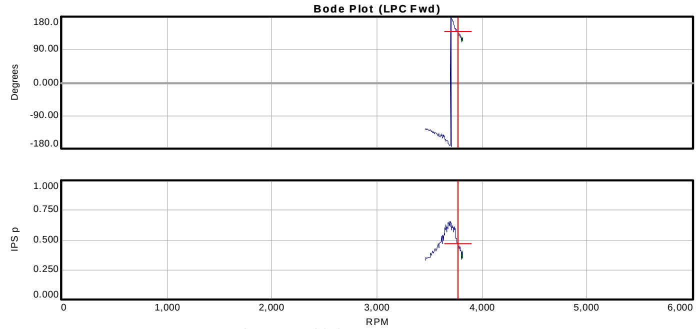

Figure 4. Bode Plot of LPC Vibration Data, Run 1

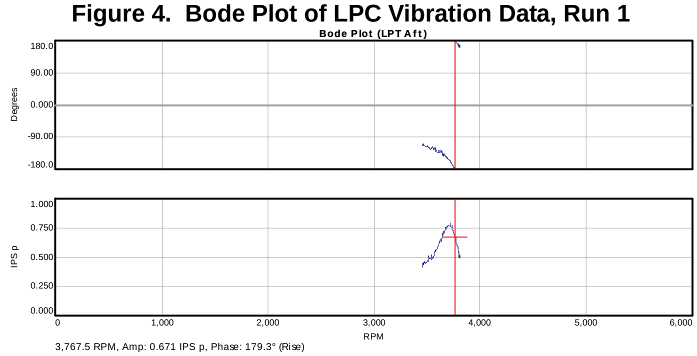

Figure 5. Bode Plot of LPT Vibration Data, Run 1

Table 2. Summary of Running Speed Vibration Data

Run	LPT Magnitude (ips)	LPT Phase (deg)	LPC Magnitude (ips)	LPC Phase (deg)
Run 1	0.67	181	0.47	218
Run 2	0.85	194	0.73	219
Run 3	0.59	186	0.40	228

Table 3. Calculated Influence Coefficients

Runs Compared	LPT Magnitude (ips/gm)	LPT Phase (deg)	LPC Magnitude (ips/gm)	LPC Phase (deg)
Runs 1 - 2	0.00421	212	0.00437	201
Runs 2 - 3	0.0028	308	0.00287	335
Runs 1 - 3	0.01212	193	0.01472	190

Several balance runs were conducted with varying weight magnitudes and locations, but reduction of LPT vibration levels remained elusive. The leading cause of this result is that the influence coefficients calculated between runs for the LPT vibrations varied widely from run to run, thus reliable and consistent balance weight placement could not be achieved. This variance in influence coefficients is attributed to (1) the fact that the gas turbine's running speed is just above a critical speed, so the magnitude and phase of vibrations are changing much more rapidly than at speeds far from the critical speed, and (2) thermal and time transient effects of aero-engine rotor construction.

Many of the components on the gas turbine rotor are held in place by centrifugal force or they are bolted in place. These components may shift slightly during speed sweeps, and the turbine needs to run at a steady speed for a period of time for these components to settle into place. For example, the age of the engine suggested that the dry lubricant in the LPC blade roots had probably deteriorated which would prevent the blades from finding the same centering locations from run to run unless operated at steady conditions for a long time. Thus, vibration data obtained from run to run during speed sweeps will probably differ slightly from data obtained after running at constant speed. Fig. 6 shows a Nyquist plot that illustrates how vibration magnitude and phase can vary significantly over at nearly constant speed prior to settling. The magnitude varies between 0.417 ips and 0.524 ips, and the phase varies from 107.4 deg to 127.1 deg while the speed is nearly constant between 3768 rpm and 3774 rpm.

Based on this experience, it was concluded that LPT vibration levels could not be effectively reduced by changing balance weights at the compressor inlet. The turbine was sent to a depot for balancing at the turbine end, where it was discovered that the LPT balance was well out of specification, and that the lubricant at the LPC blades had deteriorated.

Although the on-site compressor-end balancing effort was unsuccessful, the usefulness of the interactive balancing approach was highlighted by having the ability to manually adjust the magnitude and location of the weights and visualize the predicted effects on LPT and LPC vibration levels simultaneously. In this sense, it was possible to visually "optimize" the placement of weights very quickly. If a least squares optimization method were pursued, it would have been necessary to formulate somewhat arbitrary constraints or cost functions involving both the LPT and LPC vibrations.

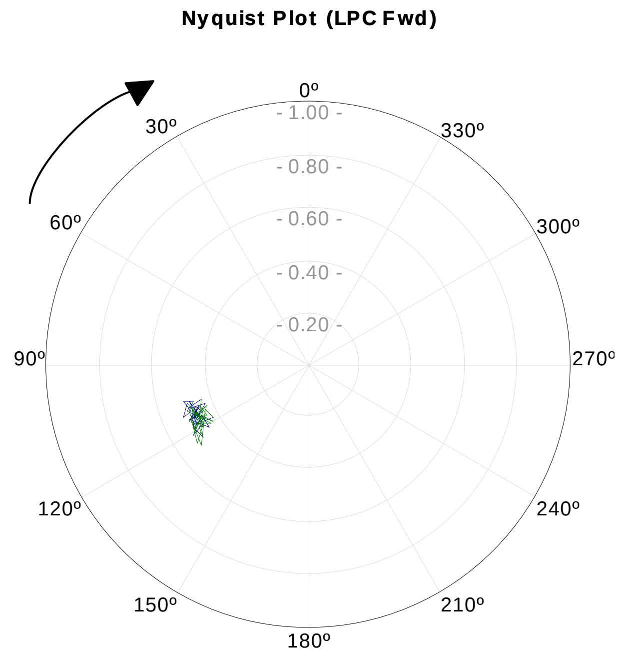

Figure 6. Nyquist Plot Showing Magnitude and Phase Drift

CASE STUDY 2: INDUCED DRAFT FAN

This example deals with the solution of a vibration problem surrounding several induced draft fans that were in process of installation at a refinery. These fans have double inlet flow impellers and mounted between bearings; they were readily balanced in cold condition using standard single plane balancing methods. Excessive vibration began to occur as the fans were brought up to their operating temperature of 600 F. Several theories were considered as to the cause of this thermal sensitivity that relate to:

a. An exceptionally tight fit hub that slips in a non-uniform way;
b. Non-uniform hub fit pressure causing the shaft to bow in response to thermal expansion;
c. Continued stress relief of an incompletely relieved shaft;
d. Non-uniform coefficient of thermal expansion (CTE) across the shaft.

Controlled thermal testing was conducted to determine repeatability. The fan was mounted in a balance machine and balanced to within 0.2 mils in ambient condition. The rotor was then heated to about 350° F and allowed to cool twice. The cold to hot differential results, shown in Fig. 7, indicate that the rotor thermal behavior is repeatable which is consistent with nonuniform CTE across the shaft diameter. The theories implicating hub slippage or insufficient stress relief were put aside as each of these phenomena would have resulted in causing the vibration to continually stepwise change with each thermal cycle and would not have been repeatable.

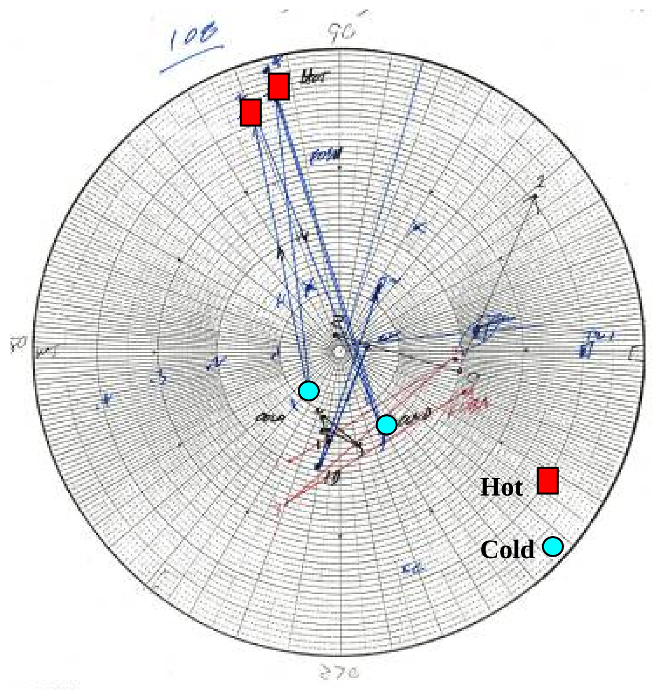

Figure 7. Fan Vibration Changes with Thermal Excursions Demonstrate Repeatability

A personal communication with Dr. Sastry Cheruvu (SwRI) [28] revealed the following supporting experience:

"The microstructure and physical properties of rotor steel depends on the composition and how it was heat treated. Years ago (in the 50s), some steel forging suppliers would quench low alloy steel steam turbine rotors in the horizontal position as the initial heat treatment, and the rotors were plagued by bowing upon subsequent service due to a nonuniform coefficient of thermal expansion. Rotor bowing was attributed to changes in microstructure during service. Segregation of alloying elements or improper heat treatment that results in non-uniform or inhomogeneous precipitation of carbides in the rotor steel can adversely affect the CTE. For example:

a) The effect of Cr content on CTE is shown by comparing the CTE (68 to 800 F) values of 1% Cr - 0.5% Mo (7.3 micro in/in/F) with 1.25% Cr - 0.5% Mo (7.7 micro in/in/F) [29]. These results show local variation of 0.25% in Cr content due to alloy segregation or inhomogeneous carbide precipitation which can result in variation of CTE by about 5%

b) The effect of Carbon on CTE is shown by comparing the CTE (70F) values of AISI 4320 (6.28 micro in/in/F) with AISI 4340 (6.83 micro in/in/F). This shows that a 0.2 % decrease in Carbon lowers CTE by about 10%."

To avoid bowing during service, all rotors should be quenched in the vertical position. Nonetheless, the rotors were delivered, and delay to wait for replacement was not acceptable from a plant start up perspective. Thus, it was decided to balance the fans to accommodate both the hot and cold conditions, with preference for minimizing vibration at the hot, long operating time condition.

Fig. 8 shows the compromise vibration results at inboard horizontal (FIBH), outboard horizontal (FOBH), and outboard axial (FOBA) locations at cold and hot condition. This plot shows that all hot vibrations can be brought to within the 0.2 ips level, but that vibration at cold might be slightly higher than desirable (0.5 ips). We could get the vibration at FOBH cold down to something more reasonable by sacrificing vibration level at hot condition; other than that, there is not much that can be done with only a single plane balance with the present shaft thermal sensitivity.

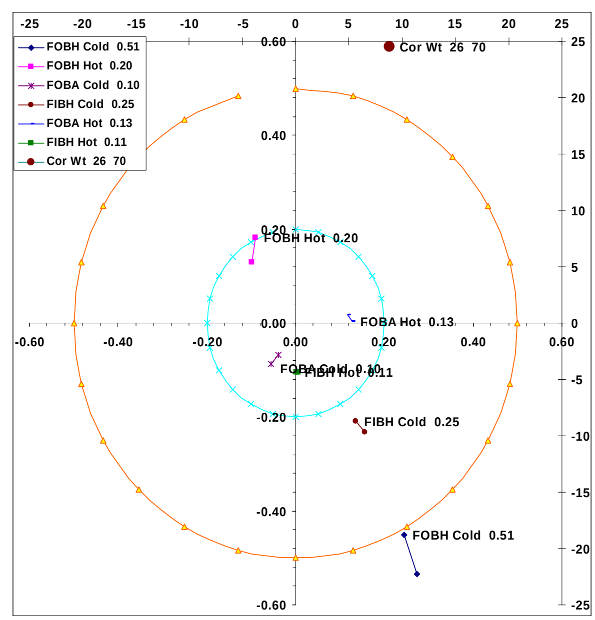

Figure 8. Fan 108 Predicted Trim Balance from Last Good Correction (145 gm @ 270 degrees)

Because of time limitations, hot condition influence coefficients could only be acquired for one fan at the time of the study. Because the other two fans were identical, however, balance corrections were calculated that should at least provide a measurable improvement for a first shot balance. While this is a reasonable approach and has worked in the past, it is not expected to be too precise as some differences in how these fans respond to unbalance should be expected. These shots should result in reduction of vibration and will provide data for a more precise balance in the next shot.

Figs. 9 and 10 show the projected results and calculated correction weights. Fan 107 - A single weight at the outboard shroud of 144 gm at 23 degrees produces 0.15 ips FOBH hot and 0.50 ips cold. Fan 105 - A single weight of 85 gm at 313 degrees produces 0.15 ips hot and 0.35 ips cold.

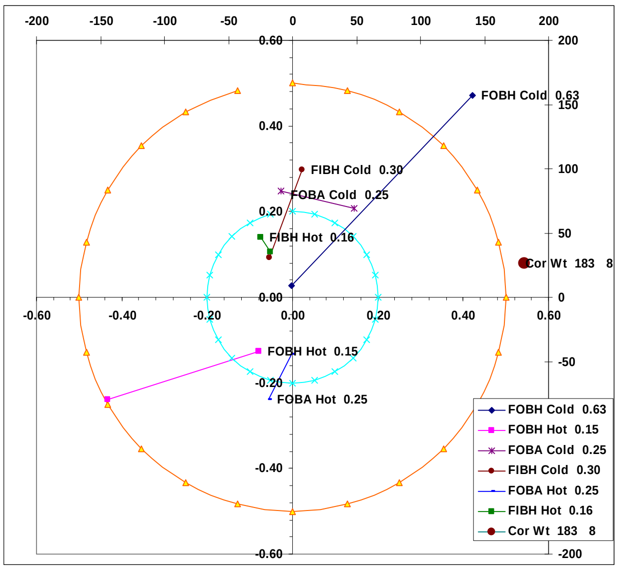

Figure 9. Fan 107 Recommended Balance Corrections and Resultant Vibration Vectors

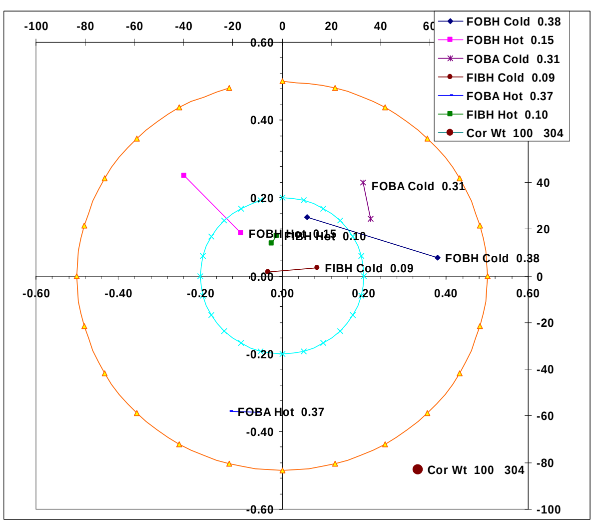

Figure 10. Fan 105 Recommended Balance Corrections and Resultant Vibration Vectors

CONCLUSIONS

This paper has presented a brief overview of balancing methods for rotating machinery and discussed an interactive implementation of equations for single plane balancing. Two case studies for balancing were presented to illustrate the usefulness of the method and provide insight into real-world balancing scenarios. In the first case study, the interactive method was applied to minimize vibrations of both LPC and LPT locations on an LM5000 gas turbine by adjusting only the weights on LPC end. This study also highlighted how error sources such as thermal transients or settling can reduce the effectiveness of balancing efforts. The second case study showed the successful use of the interactive method for optimal balancing of an industrial fan at both hot and cold operating conditions. Both examples show how the method presented in this paper can be used to interactively visualize the effects of balance weight placement on multiple conditions or points and allow the user to intuitively "optimize" balance weight placement for all conditions or points without defining a weighting function as is necessary for least squares balancing.

NOMENCLATURE

A = Initial vibration vector

B = Vibration vector after trial weight placement

M = Number of vibration measurement locations

N = Number of balance planes

W_c = Correction weight vector

W_t = Trial weight vector

$[A]$ = Column vector of vibration data for multi-plane balancing

$[W_c]$ = Column vector of correction weights for multi-plane balancing

$[\alpha]$ = Matrix of influence coefficients for multi-plane balancing

α = Influence coefficient vector

ε_m = Residual vibration vector at location *m*

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